Common Half yearly Examination -2018
XI – Mathematics – Answer Key

Part – I

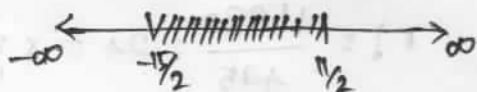| 1	②	6	①	11	④	16	④
2	④	7	②	12	③	17	④
3	④	8	④	13	②	18	②
4	①	9	③	14	④	19	②
5	④	10	③	15	①	20	①

Part – II

21. $n(A) = 3$
$n(B) = 2$
$A = \{x, y, z\}$
$B = \{1, 2\}$

22. $2|x+1| - 6 \le 7$
$2|x+1| \le 13$
$-\frac{15}{2} \le x \le \frac{11}{2}$
$x \in \left[\frac{-15}{2}, \frac{11}{2} \right]$

24. 6 distinct letters SIMPLE
∴ The number of permutation of the 6 letters all taken at a time $= 6P_6 = 6! = 720$

25. $\frac{n(n-1)}{2!} = -\frac{1}{9}$, $n = \frac{2}{3}$

$\frac{n(n-1)(n-2)}{2!} = \frac{A}{81}$

$(1+x)^{2/3} = 1 + \frac{2}{3}x - \frac{1}{9}x^2 + \frac{4}{81}x^3 + \dots$

23. LHS $= \tan(360° - 45°)\left[-\cot(360° + 45°)\right] + \cot(360° + 135°)\left[-\tan(360° + 225°)\right]$
$= [-\tan 45°][-\cot 45°] + [\tan 45°][-\tan 45°]$
$= (-1)(-1) + (-1)(-1)$
$= 2$

26.

Part - III

27. A is skew symmetric

$A^T = -A$

$$\begin{pmatrix} 0 & -1 & -2 \\ 1 & 0 & -3 \\ -2 & x^3 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -1 & 2 \\ 1 & 0 & -x^3 \\ -2 & 3 & 0 \end{pmatrix}$$

$x^3 = 3$

$x = (1/3)^{1/3} = \sqrt[3]{3}$

28. Let $y = 2^x = e^{x \log 2}$

$u = (\log 2)x$

$y = e^u$

$\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx}$

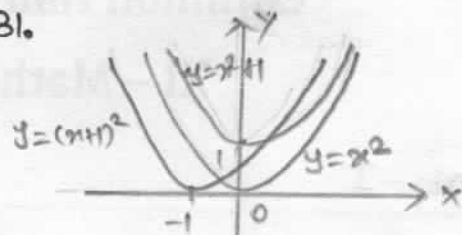$= e^u \times \log 2$

$= \log 2 \; e^{x \log 2}$

$= (\log 2) 2^x$

29. Put $+5x = y \Rightarrow x = +\dfrac{y}{5}$

$\displaystyle \lim_{x \to 0} \dfrac{e^{5x} - 1}{x} = \lim_{y \to 0} \dfrac{e^y - 1}{y/5} = 5 \lim_{y \to 0} \dfrac{e^y - 1}{y}$

$= 5(1)$

$= 5$

30. Projection of $\vec{a}$ on $\vec{b} = \dfrac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$

$= \dfrac{41}{7}$

31.

$f(x) = x^2 + 1$ causes the graph of the function $f(x) = x^2$ shifts to upward one unit.

$f(x) = (x+1)^2$ causes the graph of the function $f(x) = x^2$ shifts to left for one unit.

32. $\dfrac{x}{(x+3)(x-4)} = \dfrac{A}{(x+3)} + \dfrac{B}{(x-4)}$

$A = \dfrac{3}{7}, \; B = \dfrac{4}{7}$

$\dfrac{x}{(x+3)(x-4)} = \dfrac{3}{7(x+3)} + \dfrac{4}{7(x-4)}$

33. $8^{1 + \cos x + \cos^2 x + \cdots} = 8^2$

$1 + \cos x + \cos^2 x + \cdots = 2$

$\dfrac{1}{1 - \cos x} = 2$

$1 = 2(1 - \cos x)$

$\cos x = 1/2$

$x = \pi/3$

34. $\dfrac{nP_r}{nC_r} = r!$
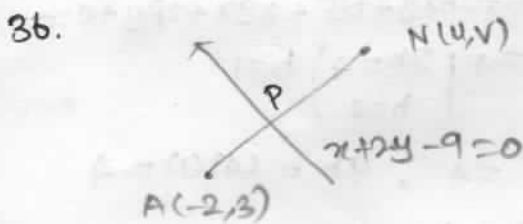
$r! = \dfrac{11880}{495} = 24 = 4!$

$nC_4 = 495$

$n = 12$

35. Let $a^{\frac{1}{x}} = b^{\frac{1}{y}} = c^{\frac{1}{z}} = k$

$a = k^x,\ b = k^y,\ c = k^z$

$b^2 = ac$ ($\because a, b, c$ are in G.P)

$k^{2y} = k^{x+z}$

$2y = x + z$

$\therefore x, y, z$ are in A.P

36.

$N(u,v)$

$x + 2y - 9 = 0$

$A(-2, 3)$

$P\left(\dfrac{-2+u}{2},\ \dfrac{3+v}{2}\right)$

P lies on $x + 2y - 9 = 0$

$u + 2v - 14 = 0 \rightarrow ①$

$\begin{pmatrix} \text{slope of} \\ AN \end{pmatrix} \begin{pmatrix} \text{slope of} \\ x+2y-9=0 \end{pmatrix} = -1$

$v - 3 = 2(u + 2)$

$2u - v + 7 = 0 \rightarrow ②$

Solve ①,②

$u = 0,\ v = 7$.

$\therefore$ Image of the point $A(-2, 3)$

is $N(0, 7)$

37. LHS $= \begin{vmatrix} 1 & a & a^2-bc \\ 1 & b & b^2-ca \\ 1 & c & c^2-ab \end{vmatrix}$

$= \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} - \begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix}$

$= \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} - \dfrac{1}{abc}\begin{vmatrix} a & a^2 & abc \\ b & b^2 & abc \\ c & c^2 & abc \end{vmatrix}$

$= \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} - \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$

$= 0$

38. $\vec{OG} = \dfrac{\vec{OA} + \vec{OB} + \vec{OC}}{3}$

$\vec{GA} + \vec{GB} + \vec{GC}$

$= \vec{OA} - \vec{OG} + \vec{OB} - \vec{OG} + \vec{OC} - \vec{OG}$

$= \vec{OA} + \vec{OB} + \vec{OC} - 3\left(\dfrac{\vec{OA} + \vec{OB} + \vec{OC}}{3}\right)$

$= \vec{0}$

39. $\dfrac{3^x - 1}{\sqrt{1+x} - 1} = \dfrac{3^x - 1}{\sqrt{1+x} - 1} \times \dfrac{\sqrt{1+x} + 1}{\sqrt{1+x} + 1}$

$= \dfrac{(3^x - 1)(\sqrt{1+x} + 1)}{x}$

$\displaystyle\lim_{x \to 0} \dfrac{3^x - 1}{\sqrt{1+x} - 1} = \lim_{x \to 0} \dfrac{3^x - 1}{x} \lim_{x \to 0} \sqrt{1+x} + 1$

$= (\log 3)(2)$

$= 2 \log 3$

$= \log 9$

40. $f'(x) = 4x + 3$

$f'(0) = 3$

$f'(-1) = -1$

$f'(0) + 3 f'(-1) = 3 - 3 = 0$

Part - IV

41. one-one : $f(x) = f(y)$

(a) $2x - 3 = 2y - 3$

$x = y$

$f(x) = f(y)$

f is one-one

onto : $y \in R$, Let $x = \dfrac{y+3}{2}$

$f(x) = 2\left(\dfrac{y+3}{2}\right) - 3 = y$

f is onto

Inverse : Let $y = 2x - 3$

$x = \dfrac{y+3}{2}$

$f^{-1}(y) = \dfrac{y+3}{2}$

Replacing y as x

$f^{-1}(x) = \dfrac{x+3}{2}$

41. (b)

$\log_{5-x}(x^2-6x+65)=2$

$\log_a b=x$

$\Rightarrow a^n=b$

$(5-x)^2=x^2-6x+65$

$x=-10$

42. (a)

$f(4)=10$

$f(-4)=-7$

$f(0)=-2$

$f(-7)=-13$

42. (b)

$\sin 2A+2\sin 2B+2\sin 2C$

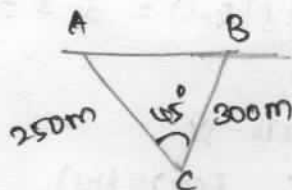$=2\sin(A+B)\cos(A-B)+2\sin 2C$

$=2\sin C\cos(A-B)+2\sin C\cos C$

$=2\sin C[\cos(A-B)-\cos(A+B)]$

$=2\sin C[2\sin A\sin B]$

$=4\sin A\sin B\sin C$

43. (a)

$\triangle ABC$, By Cosine formula

$AB^2=AC^2+BC^2-2AC\cdot BC\cdot\cos\frac{\pi}{4}$

$AB=210.95m$

43. (b)

$P(n)=2+4+6\ldots+2n=n^2+n$

Basic step: $P(1)=1+1=2$

$\therefore P(1)$ is true

Induction step: Assume that $P(k)$ is true for some +ve integer k

$P(k)=2+4+6+\ldots+2k=k^2+k$

To prove: $P(k+1)$ is also true

$[2+4+6+\ldots+2k]+2(k+1)=k^2+k+2(k+1)$

$=(k+1)^2+(k+1)$, $P(k+1)$ is true

$\therefore$ the statement $P(n)$ is true

44. (a)

$\sqrt[3]{x^3+7}=x+\frac{7}{3}\times\frac{1}{x^2}-\frac{49}{9}\times\frac{1}{x^6}+\ldots$

$\sqrt[3]{x^3+4}=x+\frac{4}{3}\times\frac{1}{x^2}-\frac{16}{9}\times\frac{1}{x^6}+\ldots$

$\sqrt[3]{x^3+7}-\sqrt[3]{x^3+4}$ till ((m)

$=\left(x+\frac{7}{3}\times\frac{1}{x^2}\right)-\left(x+\frac{4}{3}\times\frac{1}{x^2}\right)=\frac{1}{x^2}$

44. (b)

compare with

$ax^2+2hxy+by^2+2gx+2fy+c=0$

$a=4\mid 2h=4\mid b=1$

$\quad\quad\mid h=2\mid$

$h^2=4$, $ab=(4)(1)=4$

$h^2=ab$

$\therefore$ given lines are pair of parallel lines

Distance $=\sqrt{5}$

45. (a)

$\Delta=\begin{vmatrix}-2a & a+b & a+c\\ a+b & -2b & b+c\\ c+a & c+b & -2c\end{vmatrix}$

putting $b=-a$

$\Delta=0$, $(a+b)$ is a factor of Δ

^{lly} $(b+c)$, $(c+a)$ are factors of Δ.

Δ is third degree homogeneous polynomial

$\Delta=k(a+b)(b+c)(c+a)$, k is constant

$\begin{vmatrix}-2a & a+b & a+c\\ a+b & -2b & b+c\\ c+a & c+b & -2c\end{vmatrix}=k(a+b)(b+c)(c+a)$

Put $a=0, b=1, c=2 \Rightarrow k=4$

$\therefore \Delta=4(a+b)(b+c)(c+a)$

45.
(b) $5\hat{i} + 6\hat{j} + 7\hat{k} = s(7\hat{i} - 8\hat{j} + 9\hat{k}) + t(3\hat{i} + 20\hat{j} + 5\hat{k})$

Equating components

$7s + 3t = 5 \longrightarrow ①$

$-8s + 20t = 6 \longrightarrow ②$

$9s + 5t = 7 \longrightarrow ③$

Solve ①, ② $\quad s = \frac{1}{2}, \ t = \frac{1}{2}$

Sub in ③ $\quad 9(\frac{1}{2}) + 5(\frac{1}{2}) = 7$

∴ Thus one vector is a linear combination of other two vectors

∴ hence the given vectors are coplanar.

46.
(a) $A^3 = \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix}$,

$A^2 = \begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix}$

$K = 2$

46.
(b) $\vec{a} \times \vec{b} = 2\hat{i} - 2\hat{j} + \hat{k}$

$|\vec{a} \times \vec{b}| = 3$

$|\vec{c} - \vec{a}|^2 = (2\sqrt{2})^2$

$|c|^2 + |\vec{a}|^2 - 2\vec{c}.\vec{a} = 8$

$|\vec{c}| = 1$

$|(\vec{a} \times \vec{b}) \times \vec{c}| = |a \times b| \ |c| \sin 30°$

$= 3(1)(\frac{1}{2})$

$= \frac{3}{2}$

47.
(a) For any point $c \in (-1, 1)$

$\lim\limits_{x \to c} f(x) = \lim\limits_{x \to c} \sqrt{1 - x^2} = f(c)$

$\lim\limits_{x \to 1^+} f(x) = 0 = f(1)$

$\lim\limits_{x \to 1^-} f(x) = 0 = f(-1)$ ∴ Thus f is continuous on $[-1, 1]$

47.(b) $\quad y = \dfrac{\sin^{-1}x}{\sqrt{1-x^2}}$

$y\sqrt{1-x^2} = \sin^{-1}x$

Diff. w.r. to 'x'

$-xy + (1-x^2)y_1 = 1$

Again D. w.r. to 'x'

$-[xy_1 + y(1)] + (1-x^2)y_2$

$\qquad + y_1(-2x) = 0$

$-xy_1 - y + (1-x^2)y_2 - 2xy_1 = 0$

$(1-x^2)y_2 - 3xy_1 - y = 0$

$-x-$

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Way to success Teachers team

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