Way to Success Model Question Paper

A

Answer Key

(Based on new Question pattern 2019)

பிரிவு – I / SECTION – I

1	**(4)** $\boldsymbol{N}$	**11**	**(2) a ≥ g**
2	$\boldsymbol{\lvert x\rvert\lvert y\rvert}$	**12**	**(1)** $(\boldsymbol{-3,-2})$
3	இணைக்கோணங்கள் allied	**13**	**(2)** $\pm\boldsymbol{1}$
4	$\sin(-45°) = -\frac{1}{\sqrt{2}}$	**14**	**(4) 25**
5	$\frac{8!}{5!\times 2!} = 168$	**15**	**(3) 12**
6	${}^{6}P_5 = 720$	**16**	(இ) $\lim_{x\to\infty}\frac{a^x-b^x}{x} = \log\left(\frac{b}{a}\right)$ (c) $\lim_{x\to\infty}\frac{a^x-b^x}{x} = \log\left(\frac{b}{a}\right)$
7	தவறு, False	**17**	(அ) $\frac{d}{dx}\left(\frac{2}{\pi}\sin x^0\right) = \frac{1}{180}\cos x^0$ (a) $\frac{d}{dx}\left(\frac{2}{\pi}\sin x^0\right) = \frac{1}{180}\cos x^0$
8	சரி, True	**18**	(இ) $\int\sqrt{\frac{1-x}{1+x}}\,dx = \sqrt{1+x^2}+\sin^{-1}x + c$ (இ) $\int\sqrt{\frac{1-x}{1+x}}\,dx = \sqrt{1+x^2}+\sin^{-1}x + c$
9	தவறு, False	**19**	(அ) (a)
10	சரி, True	**20**	(ஈ) **(d)**

பகுதி – II / Part – II

21. $A\Delta B = (A\cup B) - (A\cap B)$

$n(A\Delta B) = n(A\cup B) - n(A\cap B)$

$= 10 - 3 = 7$

$n(\mathcal{P}(A\Delta B)) = 2^7 = 128$

22.

$3|x-2| + 7 = 19$. So that we have, $|x-2| = \frac{19-7}{3} = 4$.
Thus, we have either $x - 2 = 4$ or $x - 2 = -4$.
Therefore the solutions are $x = -2$ and $x = 6$.

23.

Let s be the length of the arc of a circle of radius r subtending a central angle θ. Then $s = r\theta$.

We have, $\theta = 15^\circ = 15 \times \frac{\pi}{180} = \frac{\pi}{12}$ radians

So that, $s = r\theta$ gives $s = 5 \times \frac{\pi}{12} = \frac{5\pi}{12}\ cm$

24. $\frac{^{(n-1)}P_3}{^{n}P_4} = \frac{1}{10}$

$\frac{(n-1)!}{(n-4)!} \times \frac{(n-4)!}{n!} = \frac{1}{10}$

$\frac{(n-1)!}{n(n-1)!} = \frac{1}{10}$

$\frac{1}{n} = \frac{1}{10}$

$n = 10$

25. Let $a^{\frac{1}{x}} = b^{\frac{1}{y}} = c^{\frac{1}{z}} = k$

$a = k^x, b = k^y, c = k^z$ ……………..(1)

Since a, b, c are in G.P, $b^2 = ac$………..(2)

$(k^y)^2 = k^x \times k^z$ [using (1) and (2)]

$k^{2y} = k^{x+z}$

$2y = x + z$

Hence, x, y and z are in A.P

26.

We factorize this equation straight away as

$$5x^2 + 6xy + y^2 = 0$$
$$5x^2 + 5xy + xy + y^2 = 0$$
$$5x(x+y) + y(x+y) = 0$$
$$(5x+y)(x+y) = 0$$

So that the lines are $5x + y = 0$, and $x + y = 0$

Alternate method: since the given equation is a homogeneous equation, divide the given equation

$5x^2 + 6xy + y^2 = 0$ by x^2

We get $5 + 6\left(\frac{y}{x}\right) + \left(\frac{y}{x}\right)^2 = 0$

Substitute $\frac{y}{x} = m$ (slope of the lines for homogenous equation)

The above equation becomes $m^2 + 6m + 5 = 0$

Factorizing, we get $(m+1)(m+5) = 0$

$\Rightarrow m = -1,\ m = -5$

$\Rightarrow \frac{y}{x} = -1,\ \frac{y}{x} = -5$

That is, the lines are $x + y = 0,\quad 5x + y = 0$

27. $\begin{bmatrix} x^2 & 1 \\ 2 & 3 \end{bmatrix} + \begin{bmatrix} 2x & 3 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ 3 & 7 \end{bmatrix}$

$x^2 + 2x = 3$

$x^2 + 2x = 3 = 0$

$(x+3)(x-1) = 0$

$x = -3, 1$

28.

$$|\vec{a}+\vec{b}| = |\vec{a}-\vec{b}|$$
$$|\vec{a}+\vec{b}|^2 = |\vec{a}-\vec{b}|^2$$
$$|\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a}\cdot\vec{b} = |\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a}\cdot\vec{b}$$
$$\Rightarrow \quad 4\vec{a}\cdot\vec{b} = 0$$
$$\vec{a}\cdot\vec{b} = 0$$

Hence $\vec{a}$ and $\vec{b}$ are perpendicular.

29. $\int cosec(5x+3)\cot(5x+3)\ dx$

$= -\frac{1}{5}\ cosec\ (5x+3) + c$

30.

$S = \{HH, HT, TH, TT\}$

$n(S) = A$

(i) Event of getting one head and one tail

$A = \{HT, TH\}$

$n(A) = 2$

P (getting one head and one tail) $= \frac{n(A)}{n(S)} = \frac{2}{4}$

$P(A) = \frac{1}{2}$

(ii) Event of getting atmost two tails

$B = \{HH, HT, TH, TT\}$

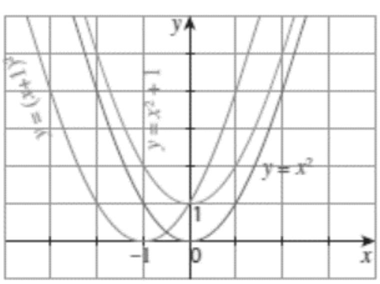
$n(B) = 4$.

P (getting atmost two tails)

$P(B) = \frac{n(B)}{n(G)} = \frac{4}{4} = \quad 1$

பகுதி – III / Part – III

31.



32.

Let $\frac{x+1}{x^2(x-1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1}$.

Then, $x+1 = Ax(x-1) + B(x-1) + Cx^2$.

When $x = 0$, we have $B = -1$ and when $x = 1$, we get $C = 2$.

When $x = -1$, we have $2A - 2B + C = 0$ which gives $A = -2$.

Thus, $\frac{x+1}{x^2(x-1)} = \frac{-2}{x} - \frac{1}{x^2} + \frac{2}{x-1}$.

33. Radius of the circle =10 ft

Included angle = $41°$

Length of arc = included angle × radius of circle

$$= 41 \times \frac{\pi}{180} \times 10$$

$$= \frac{41}{18} \times \frac{22}{7} = 7.158 \text{ ft} = 7.16 \text{ ft}$$

∴ length of arc is 7.16 ft

34.

(i) Any line is perpendicular to $3x + 4y - 12 = 0$ is $4x - 3y + k_1 = 0 \quad (k_1 \in R)$

(ii) Any line is parallel to $3x + 4y - 12 = 0$ is

$$3x + 4y + k_2 = 0 \quad (k_2 \in R)$$

35. Let G be the centroid of the ΔABC

$$\therefore \overrightarrow{OG} = \frac{\overrightarrow{OA}+\overrightarrow{OB}+\overrightarrow{OC}}{3}$$

$$\overrightarrow{GA} + \overrightarrow{GB} + \overrightarrow{GC}$$

$$= \overrightarrow{OA} - \overrightarrow{OG} + \overrightarrow{OB} - \overrightarrow{OG} + \overrightarrow{OC} - \overrightarrow{OG}$$

$$= \overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC} - 3\overrightarrow{OG}$$

$$= \overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC} - 3\left(\frac{\overrightarrow{OA}+\overrightarrow{OB}+\overrightarrow{OC}}{3}\right)$$

$$= \vec{0}$$

36.

We know that $x^n - a^n = (x-a)(x^{n-1} + x^{n-2}a + x^{n-3}a^2 + \cdots + xa^{n-2} + a^{n-1})$

$$\lim_{x\to a} \frac{x^n - a^n}{x-a} = \lim_{x\to a} \frac{(x-a)(x^{n-1} + x^{n-2}a + x^{n-3}a^2 + \cdots + xa^{n-2} + a^{n-1})}{(x-a)}$$

$$= \lim_{x\to a}(x^{n-1} + x^{n-2}a + x^{n-3}a^2 + \cdots + xa^{n-2} + a^{n-1})$$

$$= a^{n-1} + a^{n-1} + \cdots + a^{n-1} \ (n \text{ times})$$

$$\lim_{x\to a} \frac{x^n - a^n}{x-a} = na^{n-1}.$$

It is also true for any rational number n.

37.

Take $u = g(x) = x^2 + 1$ and $f(u) = \sqrt{u}$

$\therefore F(x) = (f o g)(x) = f(g(x))$

Since $f'(u) = \frac{1}{2}u^{-\frac{1}{2}} = \frac{1}{2\sqrt{u}}$ and

$g'(x) = 2x$ we get

$F'(x) = f'(g(x))g'(x)$

$= \frac{1}{2\sqrt{x^2+1}} .2x = \frac{x}{\sqrt{x^2+1}}$.

38.

$\frac{1}{\sqrt{x^2-4x+5}}$

$$x^2 - 4x + 5 = x^2 - 4x + 2^2 - 2^2 + 5$$

$$= (x-2)^2 - 4 + 5$$

$$= (x-2)^2 + 1$$

$$\int \frac{1}{\sqrt{(x-2)^2+1}} dx = \log\left|(x-2) + \sqrt{x^2 - 4x + 5}\right| + c$$

$$\left(\because \int \frac{dx}{\sqrt{x^2+a^2}} = \log\left|x + \sqrt{x^2 + a^2}\right| + c\right)$$

39.

Let S be the sample space and A be the event of getting at least two heads. Therefore, the event $\overline{A}$ denotes, getting at most one head.

$$n(S)=2^9=512,\quad n(\overline{A})=9C_0+9C_1=1+9=10$$

$$P(\overline{A})=\frac{10}{512}=\frac{5}{256}$$

$$P(A)=1-P(\overline{A})=1-\frac{5}{256}=\frac{251}{256}$$

40.

Applying $C_2 \to C_2-C_1, C_3 \to C_3-C_1$, we get

$$\text{LHS} = \begin{vmatrix} 1 & 0 & 0 \\ x & y-x & z-x \\ x^2 & y^2-x^2 & z^2-x^2 \end{vmatrix} = (y-x)(z-x)\begin{vmatrix} 1 & 0 & 0 \\ x & 1 & 1 \\ x^2 & y+x & z+x \end{vmatrix}$$

$$= (y-x)(z-x)[(z+x)-(y+x)]\ .$$

$$= (y-x)(z-x)(z-y)\ .$$

$$= (x-y)(y-z)(z-x) = \text{RHS}.$$

பகுதி – IV/ Part – IV

41. (a)

We know

$$|x| = \begin{cases} -x & \text{if} \quad x \le 0 \\ x & \text{if} \quad x > 0 \end{cases}$$

So

$$f(x) = \begin{cases} 2x-(-x) & \text{if} \quad x \le 0 \\ 2x-x & \text{if} \quad x > 0 \end{cases}$$

Thus

$$f(x) = \begin{cases} 3x & \text{if} \quad x \le 0 \\ x & \text{if} \quad x > 0 \end{cases}$$

Also

$$g(x) = \begin{cases} 2x+(-x) & \text{if} \quad x \le 0 \\ 2x+x & \text{if} \quad x > 0 \end{cases}$$

Thus

$$g(x) = \begin{cases} x & \text{if} \quad x \le 0 \\ 3x & \text{if} \quad x > 0 \end{cases}$$

Let $x \le 0$. Then

$$(g \circ f)(x) = g(f(x)) = g(3x) = 3x.$$

The last equality is taken because $3x \le 0$ whenever $x \le 0$.

Let $x > 0$. Then

$$(g \circ f)(x) = g(f(x)) = g(x) = 3x.$$

Thus $(g \circ f)(x) = 3x$ for all x.

41. (b)

$$\text{LHS} = \log 2 + 16\log\frac{16}{15} + 12\log\frac{25}{24} + 7\log\frac{81}{80}$$

$$= \log 2 + 16\,(\log 16 - \log 15) + 12\,(\log 25 - \log 24) + 7(\log 81 - \log 80)$$

$$= \log 2 + 16\log 16 - 16\log 15 + 12\log 25 - 12\log 24 + 7\log 81 - 7\log 80)$$

$$= \log 2 + 16\log 2^4 - 16\log(3\times 5) + 12\log 5^2$$
$$-12\log(2^3\times 3) + 7\log 3^4 - 7\log(2^4\times 5)$$

$$= \log 2 + 16\times 4\ \log 2 - 16\ \log 3 - 16\log 5 + 12\times 2\ \log 5$$
$$-12\ \log 2^3 - 12\ \log 3 + 7\times 4\ \ \log 3 - 7\log 2^4 - 7\log 5$$

$= \log 2 + 64\ \log 2 - 16\ \log 3 - 16 \log 5 + 24\ \log 5$

$-36\ \log 2 - 12\ \log 3 + 28\ \ \log 3 - 28 \log 2 - 7 \log 5$

$= \log 2 + 64\ \log 2 - 64 \log 2 + 28\ \ \log 3 - 28 \log 3 - 23 \log 5 + 24 \log 5$

$= \log 2 + \log 5$

$= \log_{10}(2 \times 5)$

$= \log_{10}(10)$

$= \log 10 = 1$=RHS

42. (a)

$f(-4) = -x + 4 = -(-4) + 4 = 4 + 4 = 8$
$f(1) = x - x^2 = 1 - (1)^2 = 1 - 1 = 0$
$f(-2) = x^2 - x = (-2)^2 + 2 = 4 + 2 = 6$
$f(7) = 0$
$f(0) = x^2 - x = 0^2 - 0 = 0$

42. (b)

In $\triangle ABC$, we have

(i) $\tan \frac{A-B}{2} = \frac{a-b}{a+b} \cot \frac{C}{2}$

(ii) $\tan \frac{B-C}{2} = \frac{b-c}{b+c} \cot \frac{A}{2}$

(iii) $\tan \frac{C-A}{2} = \frac{c-a}{c+a} \cot \frac{B}{2}$

We know the sine formula: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$

Now, $\frac{a-b}{a+b} \cot \frac{C}{2} = \frac{2R \sin A - 2R \sin B}{2R \sin A + 2R \sin B} \cot \frac{C}{2}$

$= \frac{\sin A - \sin B}{\sin A + \sin B} \cot \frac{C}{2}$

$= \frac{2 \cos \frac{A+B}{2} \sin \frac{A-B}{2}}{2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}} \cot \frac{C}{2}$

$= \cot \frac{A+B}{2} \tan \frac{A-B}{2} \cot \frac{C}{2}$

$= \cot \left(90^\circ - \frac{C}{2}\right) \tan \frac{A-B}{2} \cot \frac{C}{2}$

$= \tan \frac{C}{2} \tan \frac{A-B}{2} \cot \frac{C}{2} = \tan \frac{A-B}{2}$

Similarly we can prove the other two results.

43. (a)

The Sine formula is, $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}=2R$

Now, $\frac{b-c}{a}\cos\frac{A}{2}=\frac{2R\sin B-2R\sin C}{2R\sin A}\cos\frac{A}{2}$

$$=\frac{2\sin\left(\frac{B-C}{2}\right)\cos\left(\frac{B+C}{2}\right)}{2\sin\frac{A}{2}\cos\frac{A}{2}}\cos\frac{A}{2}$$

$$=\frac{\sin\left(\frac{B-C}{2}\right)\cos\left(90^\circ-\frac{A}{2}\right)}{\sin\frac{A}{2}}$$

$$=\frac{\sin\left(\frac{B-C}{2}\right)\sin\left(\frac{A}{2}\right)}{\sin\frac{A}{2}}$$

$$=\sin\left(\frac{B-C}{2}\right).$$

43. (b)

Let the given statement be P(n). i.e.,

$$P(n) = 1 + 4 + 7 + \ldots\ldots\ldots + (3n - 2) = \frac{n(3n-1)}{2}$$

Basic Step:

For n = 1, $P(1): 1 = \frac{1\,(3\times 1-1)}{2} = \frac{1\times 2}{2} = 1$

$\therefore$ P(1) is true.

Induction Step:

Assume that P(k) is true for some positive integer k. i.e.,

$$P(k): 1 + 4 + 7 + \ldots\ldots\ldots + (3k - 2) = \frac{k(3k-1)}{2} \dashrightarrow (1)$$

To Prove: P(k + 1) is also true

Proof:

$$[1 + 4 + 7 + \ldots\ldots\ldots + (3k - 2)] + [3(k + 1) - 2] = \frac{k(3k-1)}{2} + (3k + 1)$$

$$= \frac{k(3k-1)+2(3k+1)}{2}$$

$$= \frac{3k^2 - k + 6k+2)}{2}$$

$$= \frac{3k^2 + 5k+2}{2}$$

$$= \frac{(k+1)(3k+2)}{2}$$

$$= \frac{(k+1)[3(k+1)-1)}{2}$$

Thus, P(k + 1) is true, whenever P(k) is true.

Hence, by the principle of mathematical induction, the statement P (n) is true for all n ∈ N.

44. (a)

For an AP, $S_n = \frac{n}{2}[2a + (n-1)d]$
Given: $S_{10} = 52$ and $S_{15} = 77$
To find: S_{20}
$S_{10} = \frac{10}{2}[2a + (10-1)d] = 52$
$5[2a + 9d] = 52$
$10a + 45d = 52$ ……………(1)
$S_{15} = \frac{15}{2}[2a + (15-1)d] = 77$
$15[2a + 14d] = 77 \times 2$
$30a + 210d = 154$ …………(2)
$\Rightarrow 30a + 210d = 154$……….(2)
$(1) \times 3 \Rightarrow 30a + 135d = 156$ ……….(3)
$75d = -2$
$d = -\frac{2}{75}$
Substitute $d = -\frac{2}{75}$ in (1)
$10a + 45\left(\frac{-2}{75}\right) = 52$
$10a + 3\left(-\frac{2}{5}\right) = 52$
$10a - \frac{6}{5} = 52$

$10a = 52 + \frac{6}{5} = \frac{266}{5}$
$a = \frac{266}{5\times10} = \frac{133}{5\times5} = \frac{133}{25}$
Now, $S_{20} = \frac{20}{2}\left[2\left(\frac{133}{25}\right) + (20-1)\left(-\frac{2}{75}\right)\right]$
$= 10\left[\frac{266}{25} - \frac{38}{75}\right]$
$= 10\left[\frac{3\times266 - 38}{75}\right]$
$= 2 \times \frac{798 - 38}{15}$
$= 2 \times \frac{760}{15}$
$= \frac{2\times152}{3}$
$S_{20} = \frac{304}{3}$

44. (b)

Let m_1 and m_2 be the slopes of these two lines.

$$y - m_1x = 0 \text{ and } y - m_2x = 0 \quad (6.31)$$

Combined equation of these two lines is

$$(y - m_1x)(y - m_2x) = 0$$
$$y^2 - (m_1+m_2)xy + m_1m_2x^2 = 0 \quad (6.32)$$

Given that

$$ax^2 + 2hxy + by^2 = 0 \quad (6.33)$$

$$\text{Thus, } m_1 + m_2 = \frac{-2h}{b} \quad \text{and} \quad m_1m_2 = \frac{a}{b} \quad (6.34)$$

The lines perpendicular to (6.31) are

$$y + \frac{1}{m_1}x = 0 \quad \text{and} \quad y + \frac{1}{m_2}x = 0$$

The combined equation is

$$(m_1y + x)(m_2y + x) = 0$$
$$m_1m_2y^2 + (m_1+m_2)xy + x^2 = 0$$

By using (6.34) $\frac{a}{b}y^2 - \frac{2h}{b}xy + x^2 = 0$

The required equation is $ay^2 - 2hxy + bx^2 = 0$ (6.35)

45. (a)

Let $$|A| = \begin{vmatrix} 1 & x^2 & x^3 \\ 1 & y^2 & y^3 \\ 1 & z^2 & z^3 \end{vmatrix}.$$

Putting $x = y$ *gives* $$|A| = \begin{vmatrix} 1 & y^2 & y^3 \\ 1 & y^2 & y^3 \\ 1 & z^2 & z^3 \end{vmatrix} = 0 \quad (\text{since } R_1 \equiv R_2).$$

Therefore $(x - y)$ is a factor.

The given determinant is in cyclic symmetric form in x, y and z. Therefore $(y - z)$ and $(z - x)$ are also factors.

The degree of the product of the factors $(x-y)(y-z)(z-x)$ is 3 and the degree of the product of the leading diagonal elements $1 \times y^2 \times z^3$ is 5.

Therefore the other factor is $k(x^2 + y^2 + z^2) + \ell(xy + yz + zx)$.

Thus $$\begin{vmatrix} 1 & x^2 & x^3 \\ 1 & y^2 & y^3 \\ 1 & z^2 & z^3 \end{vmatrix} = [k(x^2 + y^2 + z^2) + \ell(xy + yz + zx)] \times (x-y)(y-z)(z-x).$$

Putting $x = 0$, $y = 1$ and $z = 2$, we get

$$\begin{vmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 4 & 8 \end{vmatrix} = [k(0+1+4) + \ell(0+2+0)](-1)(1-2)(2-0)$$

$$\Rightarrow \quad (8-4) = [(5k + 2\ell)](-1)(-1)(2)$$

$$4 = 10k + 4\ell \Rightarrow 5k + 2\ell = 2. \quad \dots (1)$$

Putting $x = 0, y = -1$ and $z = 1$, We get

$$\begin{vmatrix} 1 & 0 & 0 \\ 1 & 1 & -1 \\ 1 & 1 & 1 \end{vmatrix} = [k(2) + \ell(-1)](1)(-2)(1)$$

$$\Rightarrow \quad [(2k - \ell)(-2)] = 2$$

$$2k - \ell = -1. \quad \dots (2)$$

Solving (1) and (2), we get $k = 0, \ell = 1$.

Therefore $$\begin{vmatrix} 1 & x^2 & x^3 \\ 1 & y^2 & y^3 \\ 1 & z^2 & z^3 \end{vmatrix} = (x-y)(y-z)(z-x)(xy + yz + zx).$$

45. (b)

$\overrightarrow{OA} = \hat{\imath} + \hat{\jmath} + \hat{k}$; $\overrightarrow{OB} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$;

$\overrightarrow{OC} = 2\hat{\imath} - \hat{\jmath} + \hat{k}$

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$= \hat{\imath} + 2\hat{\jmath} + 3\hat{k} - \hat{\imath} - \hat{\jmath} - \hat{k}$

$= \hat{\jmath} + 2\hat{k}$

$|\overrightarrow{AB}| = \sqrt{1^2 + 2^2} = \sqrt{5}$

$\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB}$

$= 2\hat{\imath} - \hat{\jmath} + \hat{k} - \hat{\imath} - 2\hat{\jmath} - 3\hat{k}$

$= \hat{\imath} - 3\hat{\jmath} - 2\hat{k}$

$|\overrightarrow{BC}| = \sqrt{1^2 + 3^2 + 2^2}$

$= \sqrt{1 + 9 + 4}$

$= \sqrt{14}$

$\overrightarrow{CA} = \overrightarrow{OA} - \overrightarrow{OC}$

$= \hat{\imath} + \hat{\jmath} + \hat{k} - 2\hat{\imath} + \hat{\jmath} - \hat{k}$

$= -\hat{\imath} + 2\hat{\jmath}$

$|\overrightarrow{CA}| = \sqrt{1^2 + 2^2} = \sqrt{5}$

$|\overrightarrow{AB}| = |\overrightarrow{CA}| = \sqrt{5}$; $|\overrightarrow{BC}| = \sqrt{14}$

∴The given points of vector form a Isosceles Triangle.

46. (a)

Let $f(x) = \frac{x^2-9}{x^2(x^2-6x+9)} = \frac{(x-3)(x+3)}{x^2(x-3)^2}$

$f(x) = \frac{x+3}{x^2(x-3)}$

$\lim_{x\to3^-} f(x) = \lim_{x\to3^-} \frac{x+3}{x^2(x-3)}$

$= \lim_{h\to0} \frac{3-h+3}{(3-h)^2[3-h-3]}$

$= \lim_{h\to0} \frac{6-h}{(3-h)^2(-h)}$

$= \lim_{h\to0} -\frac{6-h}{h(3-h)^2}$

$= -\frac{6-0}{0(3-0)^2} = -\frac{6}{0} = -\infty$

$\lim_{x\to3^+} f(x) = \lim_{x\to3^+} \frac{x+3}{x^2(x-3)}$

$= \lim_{h\to0} \frac{3+h+3}{(3+h)^2(3+h-3)}$

$= \lim_{h\to0} \frac{6+h}{(3+h)^2 h}$

$= \frac{6+0}{(3+0)^2(0)} = \frac{6}{0} = \infty$

$\therefore f(3) \to -\infty \ as\ x \to 3^-$,

$f(3) \to \infty\ as\ x \to 3^+$

46. (b)

We know that this function is continuous at $x = 2$.

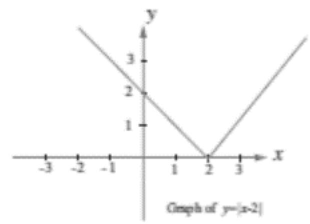
But $f'(2^-) = \lim_{x\to 2^-}\frac{f(x)-f(2)}{x-2}$

$= \lim_{x\to 2^-}\frac{|x-2|-0}{x-2}$

$= \lim_{x\to 2^-}\frac{|x-2|}{x-2} = \lim_{x\to 2^-}\frac{-(x-2)}{(x-2)} = -1$ and

$f'(2^+) = \lim_{x\to 2^+}\frac{f(x)-f(2)}{x-2}$

$= \lim_{x\to 2^+}\frac{|x-2|-0}{x-2} = \lim_{x\to 2^+}\frac{(x-2)}{(x-2)} = 1$



Since the one sided derivatives $f'(2^-)$ and $f'(2^+)$ are not equal, $f'(2)$ does not exist. That is, f is not differentiable at $x = 2$. At all other points, the function is differentiable.

If $x_0 \neq 2$ is any other point then

$$f'(x_0) = \lim_{x\to x_0}\frac{|x-x_0|}{x-x_0} = \begin{cases} 1 & \text{if } x > x_0 \\ -1 & \text{if } x < x_0 \end{cases}$$

Thus $f'(2) = \begin{cases} 1 & \text{if } x > 2 \\ -1 & \text{if } x < 2 \end{cases}$

The fact that $f'(2)$ does not exist is reflected geometrically in the fact that the curve $y = |x-2|$ does not have a tangent line at $(2, 0)$. Note that the curve has a sharp edge at $(2, 0)$.

47. (a)

Let $I = \int \tan^{-1}\left(\frac{2x}{1-x^2}\right)dx$

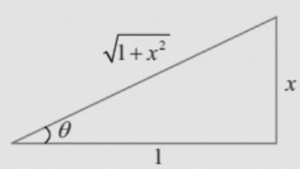
Putting $x = \tan\theta \Rightarrow dx = \sec^2\theta d\theta$

Therefore, $I = \int \tan^{-1}\left(\frac{2\tan\theta}{1-\tan^2\theta}\right)\sec^2\theta d\theta$

$= \int \tan^{-1}(\tan 2\theta)\sec^2\theta\, d\theta$

$= \int 2\theta \sec^2\theta\, d\theta$

$= 2\int (\theta)(\sec^2\theta\, d\theta)$



Applying integration by parts

$I = 2\left[\theta\tan\theta - \int \tan\theta d\theta\right]$

$= 2(\theta\tan\theta - \log|\sec\theta|) + c$

$\tan\theta = x$

$\sec\theta = \sqrt{1+x^2}$

$\int \tan^{-1}\left(\frac{2x}{1-x^2}\right)dx = 2x\tan^{-1}x - 2\log|\sqrt{1+x^2}| + c$

47. (b)

Let L be the event of selecting leap year and I be the event of selecting non leap year.

Let A be the event of getting 53 Sundays.

We have, $P(L) = \frac{1}{4}$; $P(\bar{L}) = \frac{3}{4}$

$P\left(\frac{A}{L}\right) = \frac{2}{7}$; $P\left(\frac{A}{\bar{L}}\right) = \frac{1}{7}$.

(i) $P(A) = P[(L \cap A) \cup (\bar{L} \cap A)]$

$= P(L \cap A) + P(\bar{L} \cap A)$

$= P(L)\ P\left(\frac{A}{L}\right) + P(\bar{L})\ P\left(\frac{A}{\bar{L}}\right)$

$= \frac{1}{4} \times \frac{2}{7} + \frac{3}{4} \times \frac{1}{7}$

$= \frac{5}{28}$.

(iii) $P(L \cap A) = P(L)\ P\left(\frac{A}{L}\right)$

$= \frac{1}{4} \times \frac{2}{7}$

$= \frac{1}{14}$